Additive or Multiplicative Perceptual Noise? Two Equivalent Forms of the ANCHOR Model

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Abstract

ANCHOR is an integrated memory-based scaling model that accounts for a wide range of phenomena in category rating and absolute identification. The model uses *anchors* stored in memory that serve as prototypes for each response category. The stimuli are represented by *magnitudes*. Two alternative formulations of the magnitude variability are considered: additive noise, which leads to logarithmic scales, and multiplicative noise, which leads to power scales. Both formulations are consistent with Weber's and Stevens's laws. Four variants of the ANCHOR framework systematically explore these alternative formulations. The performance of the models is evaluated against experimental data. The results show that the form of the perceptual equation is not critical for the operation of the model. Thus, the power vs. logarithmic controversy does not affect ANCHOR's central claim that human scaling performance is memory-based.

Introduction

Ever since the seminal work of Thurstone (1927) *subjective continua* occupy a prominent place in psychological theory. This notion captures in a convenient and general way two complementary aspects of the perceptual system: its systematicity and variability. A stimulus of physical intensity S gives rise to an internal *magnitude* M. Due to perceptual uncertainty, M is a random variable with non-zero variance. Its location is systematically related to the intensity S.

This paper elaborates some of these ideas within the framework of the ANCHOR model (Petrov & Anderson 2000; Petrov, 2001). The presentation rests entirely on absolute-identification data, although the model applies equally well to other scaling tasks (Petrov & Anderson, 2000, submitted). The absolute identification task is of considerable interest because it reveals some intriguing limitations of the cognitive system (Miller, 1956). Moreover, it makes direct contact with both psychophysical scaling (notably category rating) and memory (notably paired-associate learning). These are exactly the two domains that ANCHOR sets out to integrate.

The most influential model of absolute identification postulates $N-1$ criteria that partition the subjective continuum into N regions (Torgerson, 1958). When a stimulus is presented for identification, its internal magnitude falls within one of these regions and is labeled with the corresponding response. The overall accuracy is limited by uncertainties within the perceptual system ("perceptual noise") and/or in the criterion locations. When augmented with mechanisms for dynamic criterion setting this framework can account for various sequential effects (Treisman & Williams, 1984).

A criterion bisects the magnitude continuum and is very natural for binary decisions. When the number of responses increases, however, the criterion framework becomes progressively unwieldy and an alternative framework seems more appealing. Instead of emphasizing the *boundaries* between adjacent regions, it centers on the prototypes—or *anchors*—of each response category.

The anchors are magnitude-response associations that reside in memory and internalize the stimulus-response mapping required by all scaling tasks. Using the magnitude of the target stimulus as a cue, the memory system selects a single anchor that, perhaps after a minor correction, determines the response. The selection mechanism is sensitive to factors such as similarity, recency, and strength.

The anchor-based scheme offers considerable advantages (Petrov & Anderson, submitted). It is very straightforward and consistent with the introspective protocols of human observers. The growing field of memory psychophysics (Algom, 1992) provides abundant evidence that magnitude-response associations can be committed to memory and maintained over extended periods. There are also well documented sequential (e.g., Luce, Nosofsky, Green, & Smith, 1982; Lockhead & King, 1983) and context effects (e.g., Parducci & Wedell, 1986) that clearly indicate that some kind of internal state persists across trials, blocks, and even days and influences subsequent processing. Memory seems the most natural candidate to perform this function. Finally, the anchor hypothesis meshes seamlessly with the huge corpus of memory-related theory and data and in particular the ACT-R architecture (Anderson & Lebiere, 1998). ANCHOR thereby establishes connections between psychophysical scaling and a whole array of ACT-R applications.

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$S \to$ [perceptual subsystem] $\to M \to$ [central subsystem] $\to R$

Figure 1: The stimulus S maps to an internal magnitude M which in turn maps to the response R.

The link between the two theories is the construct of internal magnitude (Figure 1). It is assumed that some sensory processes, collectively referred to as *perceptual subsystem*, construct a magnitude M that encodes the intensity of the stimulus S. This magnitude is then processed within the *central subsystem* to determine the overt response R. Each subsystem can maintain an internal state that evolves in time and differs from trial to trial. The processing, therefore, is far more complex than the simple sequence suggested by the diagram. The response R depends not only on the immediate stimulus S but also, at least in principle, on all previous stimuli and responses. This gives rise to various sequential, context, transfer, and other dynamic effects.

The defining claim of the ANCHOR theory is that the bulk of the processing within the central subsystem is memory-based. This claim is supported by experimental evidence and by detailed simulations with the model (Petrov & Anderson, 2000; Petrov, 2001; Petrov & Anderson, submitted). It seems warranted, therefore, to adopt the ANCHOR characterization of the central subsystem and consider its implications for the perceptual one. This is the task we set for ourselves in this article.

The next section presents ANCHOR first in general terms and then with specific equations. Building on this foundation, subsequent sections discuss two alternative forms of the perceptual equation. Then an identification experiment is reported and the alternative versions of the model are fitted to the data.

Main Principles of ANCHOR

Internal Magnitude Continuum. Each stimulus induces a subjective magnitude M. It is this internalized quantity that can be committed to memory and compared against other magnitudes.

Content-Addressable Memory. The second principle postulates content-addressable memory involving these magnitudes. In particular, it is possible to establish associations between a magnitude and the label of a response category. These *anchors* substantiate the mapping between magnitudes (and hence the stimuli represented by them) and responses. When a new target magnitude is produced by the perceptual subsystem, the memory fills in the corresponding response label. This completion process is stochastic and depends on two factors: (a) the location of the target magnitude with respect to the various anchors in memory and (b) the frequency and recency of use of each response category. The latter factor is captured by the *base-level activations* (or *biases*) of the anchors. These activations play a very important role in the theory and make direct contact with many memory-related phenomena.

Explicit Correction Strategies. Because the memory system is noisy and prone to biases, it is not guaranteed to provide on each trial the anchor that best matches the target magnitude. The verbal protocols of human observers suggest that they are aware of the unreliability of their "first guesses" and adopt explicit correction strategies. Consequently, the third ANCHOR principle provides for such explicit corrections. Phenomenologically, an introspective report of a trial might go like this, "I see the stimulus... It looks like a *7*... No, it's too short for a *7*; I'll give it a *6*." Such increments and decrements have far-reaching implications and are vital for the stability of the overall system, especially in the absence of feedback.

Obligatory Learning. So, the stimulus has been encoded, matched against anchors, and a response has been produced. Is this the end of the trial? *No*, according to the fourth ANCHOR principle. Two learning mechanisms update the internal state of the model: the base-level activations and locations of the anchors. All changes are incremental and give rise to various dynamic effects.

The Perceptual Subsystem

ANCHOR uses a simplified generic formulation of the perceptual subsystem that still takes into account the fundamental empirical constraints imposed by Weber's and Stevens's laws. The whole subsystem is modeled by a single equation describing the distribution of magnitudes as a function of the stimulus intensity S. It abstracts away factors such as attention, habituation, Gestalt, etc. They can be included in the future without disrupting the rest of the theory.

Weber's Law. One empirical constraint that cannot be neglected by any credible scaling system is that the *difference threshold* ΔS tends to be proportional to S over much of the dynamic range of the stimulus attribute. Thus the ratio of the two—the *Weber fraction*—is approximately constant for a given perceptual modality:

$$\Delta S/S = k = const \qquad (1)$$

Stevens's Law. The other major empirical regularity comes from a vast array of magnitude estimation and category rating studies (Stevens, 1957, 1975). For intensive (or *prothetic*) continua the average rating R varies approximately as a power function of the stimulus intensity S:

$$R = aS^n \qquad (2)$$

Both Weber's and Stevens's laws are subject to qualifications and various alternative formulations have been proposed (e.g., Ekman, 1959; Norwich & Wong, 1997). Most of them deal with deviations near the low absolute threshold and can be put aside for our present purposes.

Additive Noise Equation. The standard interpretation of Weber's law is that the subjective magnitude M is proportional to the logarithm of the stimulus. Assuming equal variance (*Fechner's postulate*), this explains the progressively poorer discriminability at higher intensity levels. Equation 3 formalizes these ideas. In it, a is an arbitrary conversion factor and ε_p is a Gaussian deviate with mean zero and variance scaled by the free parameter $\sigma_p = const$. This perceptual noise makes the magnitude M a random variable too.

$$M = a\,(\log S + \sigma_p \varepsilon_p) \tag{3}$$

Multiplicative Noise Equation. It is possible, however, that the standard deviation of each magnitude distribution grows in proportion to its mean (*Ekman's law*, 1959). The spacing among the means can thus be less compressive than the logarithm in Eq. 3 and still produce poorer discriminability at higher intensities. In fact, it has been shown mathematically that when the centers of the magnitude distributions vary as a power function of the stimuli, Ekman's law implies Weber's law and vice versa (Norwich & Wong, 1997; Petrov & Anderson, submitted). This leads to Equation 4, in which n is the exponent from Stevens's power law (Eq. 2) and k_p is a dimensionless coefficient of proportionality. The noise ε_p has zero mean and unit variance as in Eq. 3.

$$M = aS^n(1 + k_p \varepsilon_p) \tag{4}$$

In summary, we have two alternative equations, one with additive and the other with multiplicative perceptual noise, that are equally consistent with the two foremost empirical regularities in the psychophysical literature.

Faced with this underdetermined situation the theoretician has a choice. We have no strong commitments on this issue, although all ANCHOR simulations reported so far (Petrov, 2001; Petrov & Anderson, 2000, submitted) use Equation 4. Our goal in the present paper is to investigate whether this particular choice limits the applicability of our earlier results. To that end, we compare the behavior of the model under additive and multiplicative noise, everything else being equal. Before embarking on this project, however, a brief description of the other ANCHOR mechanisms is in order.

The Central Subsystem

The model maintains an anchor for each response category. The *location* L_i of each anchor i represents the current estimate of the prototypical member of the corresponding category. When a target magnitude M is presented for identification, it acts as a memory cue and the anchors compete to "match" this target. Due to memory fluctuations, the processing on each trial depends on *anchor magnitudes* A_i, which are noisy versions of the locations L_i. For consistency, the memory noise in the model has the same form as its perceptual counterpart. Thus, Equation 5 and 3 form a pair, and similarly Equations 4 and 6. The standard deviation of the additive memory noise is $a\,\sigma_m$ and the coefficient of the multiplicative noise is k_m. Again, ε_m is Gaussian.

$$A_i = L_i + a\,\sigma_m \varepsilon_m \tag{5}$$

$$A_i = L_i\,(1 + k_m \varepsilon_m) \tag{6}$$

A selection mechanism determines, stochastically, a single anchor on each trial. The outcome of the competition is described by two equations in the model. Equation 7 produces *goodness scores* G_i and the "softmax" Equation 8 converts them into selection probabilities P_i:

$$G_i = -\,|M - A_i| + HB_i \tag{7}$$

$$P_i = \frac{\exp(G_i/T)}{\sum_j \exp(G_j/T)} \tag{8}$$

Each goodness score G_i is a sum of two terms: similarity $-\,|M - A_i|$ and history HB_i. The first is simply the negation of the mismatch between the target magnitude M and the anchor magnitude A_i. The second term reflects the *base-level activation* B_i of the anchor, weighted by a parameter H. It does not depend on the target at all. The "temperature" parameter T controls the degree of non-determinism in the selection process.

The memory system is noisy and prone to biases. Therefore it is not guaranteed to provide the anchor that best matches the target. The correction mechanism attempts to compensate for that. It compares the target magnitude M and the anchor magnitude A to determine the discrepancy $D = M - A$. If the latter is less than some cutoff value c, the response associated with the anchor is chosen as the final response on the trial. Otherwise the anchor response is corrected by ± 1 or occasionally even ± 2 depending on the algebraic difference D. The respective cutoffs are $\pm c$ and $\pm 3c$. The final response R is the sum of the anchor label and the correction, clipped between the lowest and highest valid category if needed.

The cutoff parameter c is chosen so that the corrections are conservative—substantial discrepancy D is required to trigger any changes. The memory-related effects introduced during the anchor selection process thus persist, albeit attenuated, and produce sequential and context effects in the responses.

At the end of the trial, feedback indicating the correct response is typically provided in absolute identification experiments. In category rating without feedback, the model uses its own response as the best available estimate. Either way, exactly one anchor is considered "used" on that trial and its location L is updated according to Equation 9, which is a form of competitive learning. The new location is pulled towards the target magnitude M, thereby improving the chances that the same anchor will match this target in the future. This tends to promote consistency but has other consequences as well, notably context effects (Petrov & Anderson, submitted).

$$L^{(t+1)} = (1-\alpha)L^{(t)} + \alpha M^{(t)} \tag{9}$$

In the long run, the location of each anchor becomes a running average (exponentially discounted by the learning rate α) of the magnitudes of all stimuli classified under the associated response category. Therefore the anchors represent true prototypes.

In contrast to the competitive learning mechanism, the base-level learning Equation 10 updates the availability of every anchor on each trial. The formula is not transparent and can be discussed only briefly here. It is an approximate and parameter-free version of the base-level learning equation in ACT-R (Anderson & Lebiere, 1998, p. 124). The availability B of a given anchor reflects the frequency and recency of its use. The formula disregards the detailed history and retains only three critical pieces of information: the lag since the most recent use t_{last}, the total time since the creation of the anchor t_{life}, and the overall number of uses n.

$$B = \log\left[t_{last}^{-0.5} + \frac{2(n-1)}{\sqrt{t_{life}} - \sqrt{t_{last}}}\right] \tag{10}$$

Qualitatively, Equation 10 captures three important aspects of memory dynamics: sharp transient boost immediately after use, gradual buildup of strength with frequent use, and gradual decay in the absence of use.

Identification Experiment

To evaluate the performance of the ANCHOR model under the alternative noise formulations, we use the data set from an absolute identification experiment reported in full detail in (Petrov & Anderson, submitted). Only a small subset of the data is sufficient for our present purposes and is described below.

Method. The stimuli were pairs of dots presented at randomized locations on a monitor. The independent variable was the distance between the dots. Only 9 stimulus lengths were involved: 275, 325, 375, ..., 675 pixels (275 pix ≈ 88 mm ≈ 8.4 deg. visual angle; 675 pix ≈ 216 mm ≈ 20 d.v.a). The imaginary segment formed by the dots was always horizontal. 24 naïve observers were instructed that there were 9 stimuli and 9 responses and that their task was to identify "the distance between the dots" by pressing a key from *1* to *9*. Each observer completed 450 trials with feedback. The stimulus presentation frequencies were non-stationary in order to induce context and transfer effects (see Petrov & Anderson, submitted, for details). The presentation schedule was counterbalanced so that each stimulus appeared exactly 50 times.

Results. The experiment yielded a wealth of data and replicated all classical absolute-identification phenomena falling within its scope. These included: limited information capacity (Miller, 1956), various sequential effects, repetition effect, edge effect, and practice effect. An unexpected assimilative context effect was also found (Petrov & Anderson, submit.).

The linear correlation coefficient between stimuli and responses is extremely high ($r > 0.92$ for all observers). This suggests a linear relationship and can be interpreted as a power law with $n = 1$ (Eq. 2). This replicates the robust finding that the exponent for line length is very close to 1.0 (Stevens, 1975).

Three empirical profiles are singled out as our current modeling targets. They are plotted with "–×–" symbols in Figure 2. The top panel shows the overall probability of correct identification for each of the nine stimuli. An edge effect is clearly visible.

The elevated accuracy near the edges could stem from the simple fact that there are fewer possibilities for mistake there. The *inter-stimulus discriminability* $d'_{i,i+1}$ is a better measure of the identification performance (Luce et al., 1982). It is calculated from the $S \times R$ probability matrix for each of the 8 inter-stimulus boundaries. Whenever S_{i+1} is presented, all responses $\geq i+1$ are considered "hits" and those $\leq i$ "misses." On the other side of the boundary, on trials with S_i the responses $\geq i+1$ are "false alarms" and $\leq i$ "correct rejections." The discriminability d' is then computed in the usual way, separately for each participant. The middle panel in Figure 2 plots the group average.

The asymmetry in the d' profile suggests that short distances are more discriminable than long ones. This finding is directly related to Weber's law (Eq. 1) and hence to the issues of main interest here.

Each stimulus elicits a whole distribution of responses. The third panel in Figure 2 plots the standard deviations of these distributions. The profile seems to increase but not by much: all values vary between $\sigma_2 = 0.63$ and $\sigma_8 = 0.81$ (save the flanks, which are contaminated by edge effects).

Let us assume temporarily, as Stevens (1957) once did, that the overt responses R are direct reports of the internal magnitudes M. Then the additive noise Equation 3 would predict a flat deviation profile for all stimuli. The multiplicative Equation 4 with $n = 1$, on the other hand, predicts proportionality.

On the surface, the empirical profile looks like a compromise between these two extremes. Such conclusion, however, is unwarranted because the re-

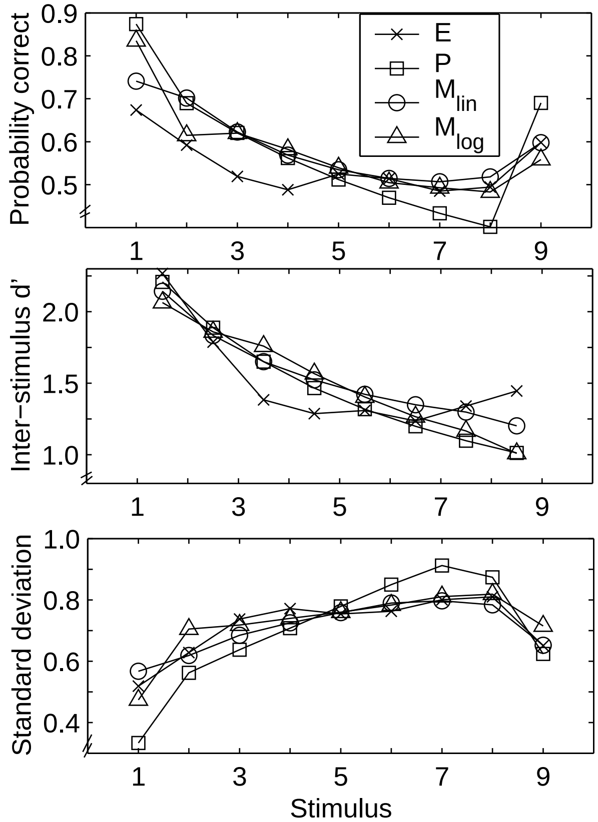

Figure 2: Empirical profiles (E) and model fits for the perceptual mechanism alone (P) and the full model with linear (M_{lin}) and logarithmic (M_{log}) magnitude scales. Overall accuracy (top), discriminability d' for each inter-stimulus boundary (middle), and standard deviation of responses (bottom).

sponses are only indirectly related to the internal magnitudes. Moreover, the task involves feedback. The deviation data must be interpreted in a framework that takes the $M \rightarrow R$ transition in Figure 1 into account. Thus we turn to the ANCHOR model.

Model Fits and Simulations

We experiment with four variants of the model—denoted $\mathcal{P}_{lin}$, $\mathcal{P}_{log}$, $\mathcal{M}_{lin}$, and $\mathcal{M}_{log}$ below. The two "perception only" variants $\mathcal{P}$ bypass most central mechanisms in order to highlight the perceptual subsystem. The $\mathcal{M}$ variants engage all mechanisms. In particular, $\mathcal{M}_{lin}$ is synonymous with the standard ANCHOR model (Petrov & Anderson, submitted).

The multiplicative noise Equation 4 defines a linear magnitude scale when $n = 1$. This is the basis of model $\mathcal{P}_{lin}$. The conversion factor a is arbitrarily set to $a_{lin} = 0.001$ so that stimulus S_5, which is 475 pixels long, produces magnitudes centered on $M = 0.475$. The coefficient k_p is the only free parameter of this model. The nine anchors are optimally placed (at the images of the stimuli) and the response is always based on the anchor that best matches the target. This is equivalent to a Thurstonian system with fixed criteria (Torgerson, 1958).

Table 1: Root mean squared errors of the fits of the models described in the text to the empirical profiles.

Profile	$\mathcal{P}_{lin}$	$\mathcal{P}_{log}$	$\mathcal{M}_{lin}$	$\mathcal{M}_{log}$
Accuracy	.10	.10	.06	.07
Discriminability	.21	.21	.17	.25
Variability	.10	.09	.03	.04

Model $\mathcal{P}_{log}$ is based on the additive noise Equation 3 and hence a logarithmic magnitude scale. In an effort to make the simulations as comparable as possible, the conversion factor a in this case is set to $a_{log} = 0.0771$. Thus the image of S_5 is again $M = 0.475$. The anchors are placed at the logarithmic images of the stimuli. Everything else is as in $\mathcal{P}_{lin}$. The standard deviation σ_p is a free parameter.

Model $\mathcal{M}_{lin}$ upgrades $\mathcal{P}_{lin}$ with all central mechanisms. The memory noise is multiplicative (Eq. 6). This amounts to the standard ANCHOR model and provides default values for many parameters (Petrov & Anderson, submitted). It is convenient to formulate them in *category-size units* δ. The distance between any two adjacent categories on the linear scale is $\delta_{lin} = 0.050$. The default history parameter is $H = 0.100 = 2\delta$ (Eq. 7); temperature $T = 0.050 = \delta$ (Eq. 8); rate $\alpha = 0.3$ (Eq. 9). Three free parameters remain: k_p, k_m, and the cutoff c.

Finally, model $\mathcal{M}_{log}$ explores an additive-noise version of ANCHOR. Equations 3 and 5 replace 4 and 6, respectively. Everything else remains the same. The new category-size unit is estimated as the geometric mean of the eight intervals on the scale. For $a_{log} = 0.0771$ this yields $\delta_{log} = 0.0084$. The default parameters can now be converted: $H = 2\delta = 0.0167$, $T = \delta = 0.0084$, $\alpha = 0.3$. Model $\mathcal{M}_{log}$ thus has three free parameters: σ_p, σ_m, and c.

Each model is fitted to the empirical d' profile via least mean squares. The optimal parameters are as follows. $\mathcal{P}_{lin}$: $k_p = 0.076$. $\mathcal{P}_{log}$: $\sigma_p = 0.076$. $\mathcal{M}_{lin}$: $k_p = 0.031$, $k_m = 0.046$, $c = 0.75\,\delta_{lin}$. $\mathcal{M}_{log}$: $\sigma_p = 0.041$, $\sigma_m = 0.050$, $c = 0.50\,\delta_{log}$.

Next, we generate predictions from the models and compare them against the three empirical profiles (Fig. 2). The predictions for models $\mathcal{P}_{lin}$ and $\mathcal{P}_{log}$ can be calculated directly from the corresponding perceptual equation. For the full models we must resort to simulations. Both models are run 10 times on each of the 24 stimulus sequences shown to the human observers. The responses are then analyzed in exactly the same way as the empirical data.

Figure 2 plots the simulated profiles and Table 1 reports the associated root mean squared errors.

The two "perception only" variants are nearly equivalent. In fact, their profiles are so close to each other that are plotted together in Figure 2. This is consistent with their mathematically proven equivalence with respect to Weber's law and suggests that the equivalence extends to the absolute identification task as well. (The proofs are for the 2AFC task.) Remarkably, the response variability profiles (bottom panel) of $\mathcal{P}_{lin}$ and $\mathcal{P}_{log}$ are very similar too, with only minor discrepancies at the edges. Thus, even though the noise on the *magnitude* continua are qualitatively different in the two models, the variability of the overt *responses* is the same. This is explained by the compensatory spacing of the anchors. The predicted profile, however, is too steep in comparison with the experimental data.

The full models $\mathcal{M}_{lin}$ and $\mathcal{M}_{log}$ are superior to their simpler counterparts. In general, the central subsystem tends to redistribute resources among the anchors, thereby reducing the steepness and asymmetry of all three profiles. This improves the fits as the empirical profiles tend to be quite level (barring the edge effects). The only feature that all four models fail to reproduce is the upward turn at the right edge of the d' profile (the *bow effect*). Model $\mathcal{M}_{lin}$ is the least discrepant from the experimental data in this region, which explains its reduced error ($rmse = 0.17$) relative to the other models.

Finally, we come to the comparison of greatest interest: $\mathcal{M}_{lin}$ versus $\mathcal{M}_{log}$. The quantitative fits in Table 1 show that the standard ANCHOR model ($\equiv \mathcal{M}_{lin}$) performs slightly better. The additive-noise variant, however, does not lag far behind and its profiles are qualitatively very similar. Moreover, $\mathcal{M}_{log}$ competes with a handicap as all default parameters have been fine-tuned within the multiplicative framework. Also, the correction mechanism assumes uniform category sizes (on the magnitude continuum) in accordance with the assumptions of the linear model. Under the logarithmic reformulation, this tends to generate corrections that are too aggressive for the shortest stimuli and too conservative for the longest ones. It seems likely that model $\mathcal{M}_{log}$, which already achieves very good fit, can be fine-tuned to the extent to which $\mathcal{M}_{lin}$ is. In light of all these considerations the final outcome of the competition appears to be a tie.

Conclusions

Our results show that the distinction between additive and multiplicative noise does not significantly affect the ability of the ANCHOR model to account for the data from the identification experiment. Consequently, the controversy between logarithmic and power-based sensory scales that has dominated the psychophysical literature since Stevens's original paper (1957) cannot detract from ANCHOR's primary goal—to explore the hypothesis that the transition between magnitudes and responses is memory-based. The present paper contributes to this goal by showing that the particular form of the perceptual equation is not critical for the operation of the model. This suggests that the successful accounts provided by the memory hypothesis for over a dozen sequential, context, and other dynamic effects are not dependent on this issue either.

References

Algom, D. (1992). Memory psychophysics: An examination of its perceptual and cognitive prospects. In D. Algom (Ed.), *Psychophysical approaches to cognition*. Amsterdam: Elsevier.

Anderson, J. R., & Lebiere, C. (1998). *The atomic components of thought*. Hillsdale, NJ: LEA.

Ekman, G. (1959). Weber's law and related functions. *The Journal of Psychology, 47*, 343–352.

Lockhead, G., & King, M. (1983). A memory model of sequential effects in scaling tasks. *Journal of Experimental Psychology: HPP, 9*, 461–473.

Luce, R. D., Nosofsky, R., Green, D. M., & Smith, A. (1982). The bow and sequential effects in absolute identification. *Perc. & Psychophysics, 20*, 49–54.

Miller, G. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Rev, 63*, 81–97.

Norwich, K., & Wong, W. (1997). Unification of psychophysical phenomena: The complete form of Fechner's law. *Perc. & Psychophys., 59*, 929–940.

Parducci, A., & Wedell, D. (1986). Category effects with rating scales: Number of categories, number of stimuli, and method of presentation. *Journal of Experimental Psychology: HPP, 12*, 496–516.

Petrov, A. (2001). Fitting the ANCHOR model to individual data: A case study in Bayesian Methodology. *Proceedings of the Fourth Intern. Conf. on Cognitive Modeling*. Hillsdale, NJ: LEA.

Petrov, A., & Anderson, J. R. (2000). ANCHOR: A memory-based model of category rating. *Proceedings of the 22 Annual Conf. of the Cognitive Science Society* (pp. 369–374). Hillsdale, NJ: LEA.

Petrov, A., & Anderson, J. (submitted). ANCHOR: An integrated memory-based scaling model.

Stevens, S. (1957). On the psychophysical law. *Psychological Review, 64*, 153–181.

Stevens, S. (1975). *Psychophysics: Introduction to its perceptual, neural and social prospects*. Wiley.

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review, 34*, 273–286.

Torgerson, W. S. (1958). *Theory and methods of scaling*. New York, Wiley.

Treisman, M., & Williams, T. (1984). A theory of criterion setting with an application to sequential dependencies. *Psychological Review, 91*, 68–111.