Bayesian Method for Repeated Threshold Estimation

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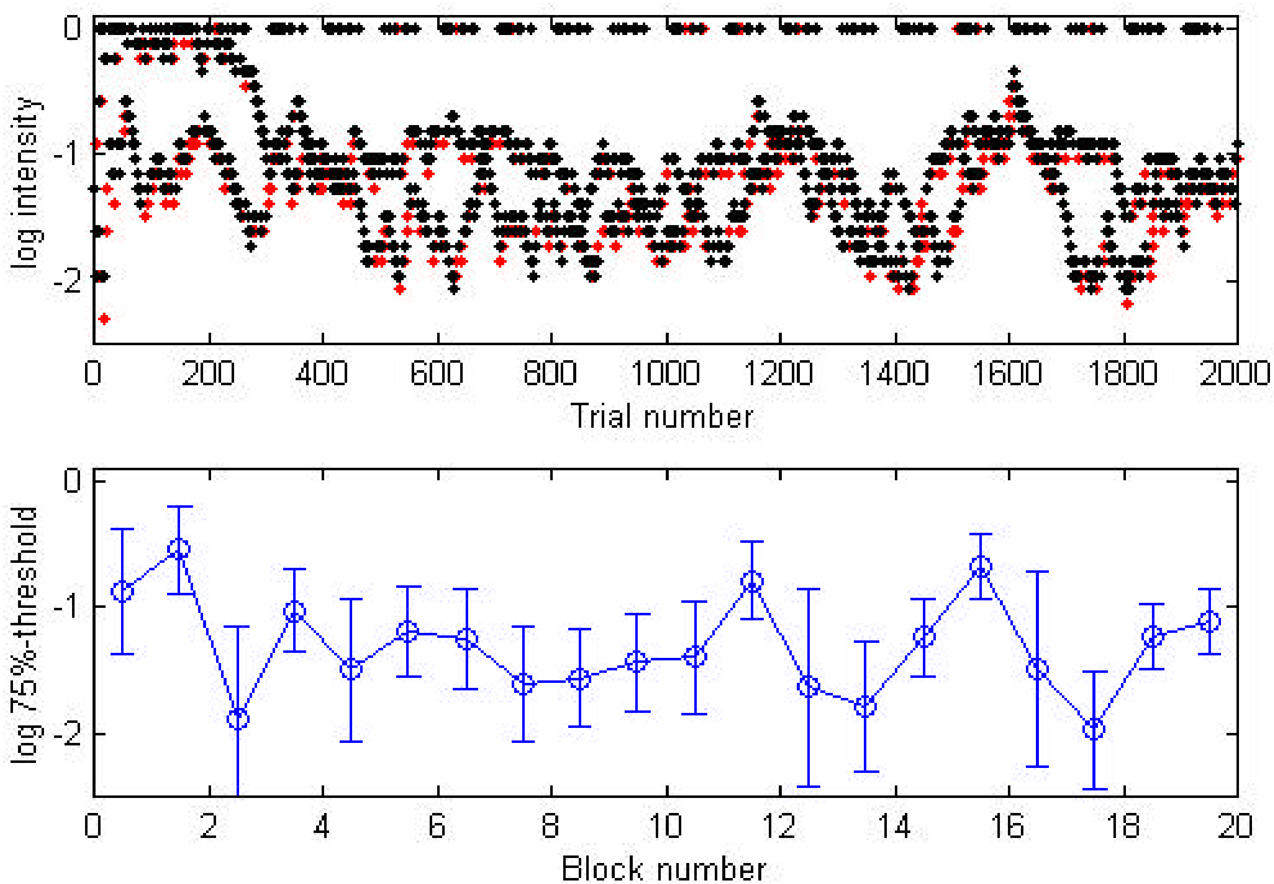
Motivation: Perceptual Learning

- Non-stationary thresholds
- Dynamics of learning is important
- Must use naïve observers
- Low motivation → high lapsing rates
- Slow learning → many sessions

- Large volume of low-quality binary data

Objective: Data Reduction



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Isn’t This a Solved Problem?

- Up/down (Levitt, 1970)
- PEST (Taylor & Creelman, 1967)
- BEST PEST (Pentland, 1980)
- QUEST (Watson & Pelli, 1979)
- ML-Test (Harvey, 1986)
- Ideal (Pelli, 1987)
- YAAP (Treutwein, 1989)
- and many others…

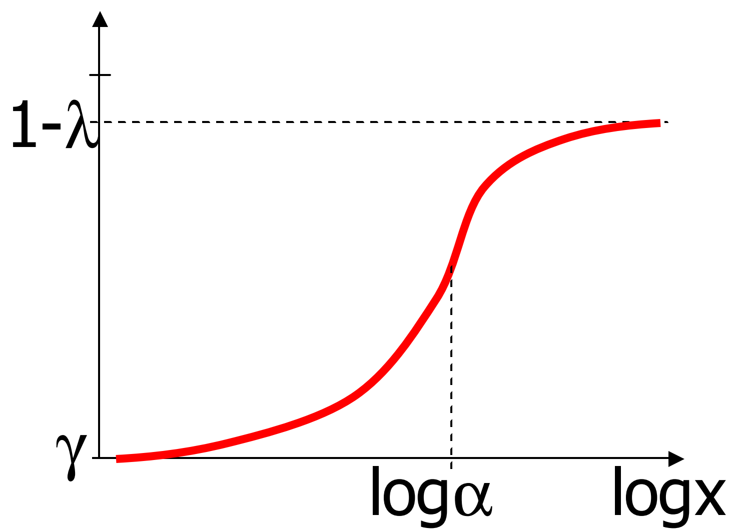
We Solve a Different Problem

- Standard methods:
  - Adaptive stimulus placement
  - Stopping criterion
  - Threshold estimation
- Our method:
  - Threshold estimation
  - Integrate information across blocks

Weibull Psychometric Function

$$W(x;a,b) = 1 - \exp(-\exp((\log x - \log a)b))$$

$$P(x;a,b,g,l) = g + (1 - g - l)W(x;a,b)$$



- Threshold log α
- Slope β
- Guessing rate γ
- Lapsing rate λ

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Two Kinds of Parameters

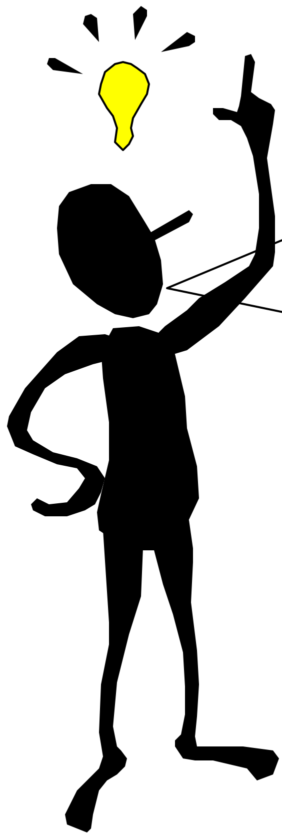
- Threshold log α
- Slope β
- Guessing rate γ
- Lapsing rate λ

Parameters of interest θ

Nuisance parameters ϕ

The nuisance parameters are harder to estimate but change more slowly than the threshold parameter.

Get the Best of Both Worlds



Use long data sequences to constrain the nuisance parameters; use short sequences to estimate the thresholds.

Joint Posterior of θ_k, ϕ

$$p(q_k, f \mid \mathbf{y}_k; \mathbf{y}_1 \ldots \mathbf{y}_{k-1}, \mathbf{y}_{k+1} \ldots \mathbf{y}_n) =$$

$$\underbrace{p(\mathbf{y}_k \mid q_k, f)}_{\text{Likelihood of current data}} \underbrace{\underbrace{p(q_k)\, p(f)}_{\text{Priors}} \underbrace{\prod_{i \neq k} \int p(q_i)\, p(\mathbf{y}_i \mid q_i, f)\, dq_i}_{\text{Information about } \phi \text{ extracted from the other data sets}}}_{\text{Modified prior for the current block}}$$

Two-Pass Algorithm

- Pass 1: for each block i, calculate

$$p(f \mid \mathbf{y}_i) = \int p(q)\, p(\mathbf{y}_i \mid q, f)\, dq$$

- Pass 2: for each block k, calculate

$$p(q_k, f \mid \mathbf{y}_k) = p(\mathbf{y}_k \mid q_k, f)\, p(q_k)\, p(f) \prod_{i \neq k} \int p(f \mid \mathbf{y}_i)$$

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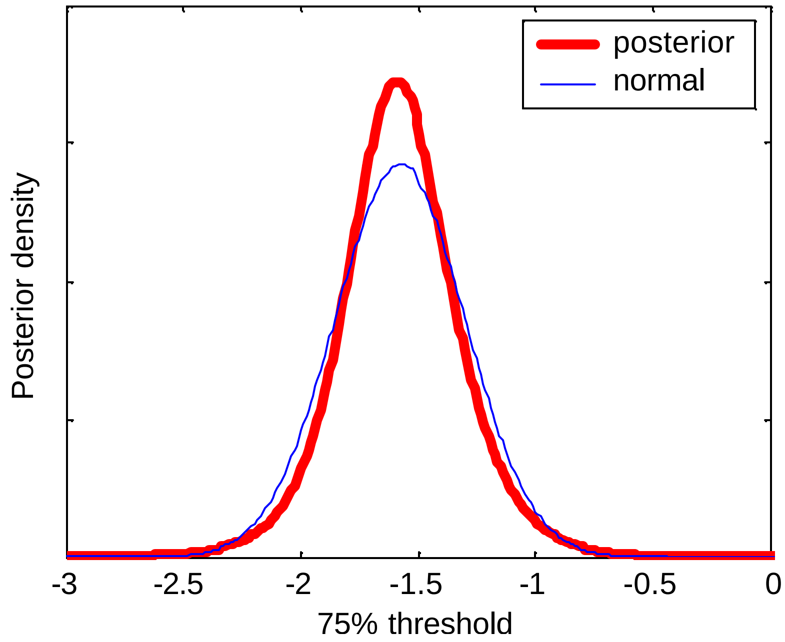
Posterior Thresholds

$$p(T_k) = \int P^{-1}(.75; q_k, f)\, p(q_k, f \mid \mathbf{y}_k)\, dq_k\, df$$



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Some Details

- Vaguely informative priors:

$$p(\log a) \propto \mathrm{N}(m_a, s_a)$$
$$p(b) \propto \mathrm{N}(m_b, s_b)$$
$$p(l) \propto \mathrm{Beta}(a_l, b_l)$$

- Implemented on a grid: $\log\alpha \times \beta \times \lambda$
- Assume γ=.5 for 2AFC data
- MATLAB software available at http://www.socsci.uci.edu/~apetrov/

Simulation 1: Stationary

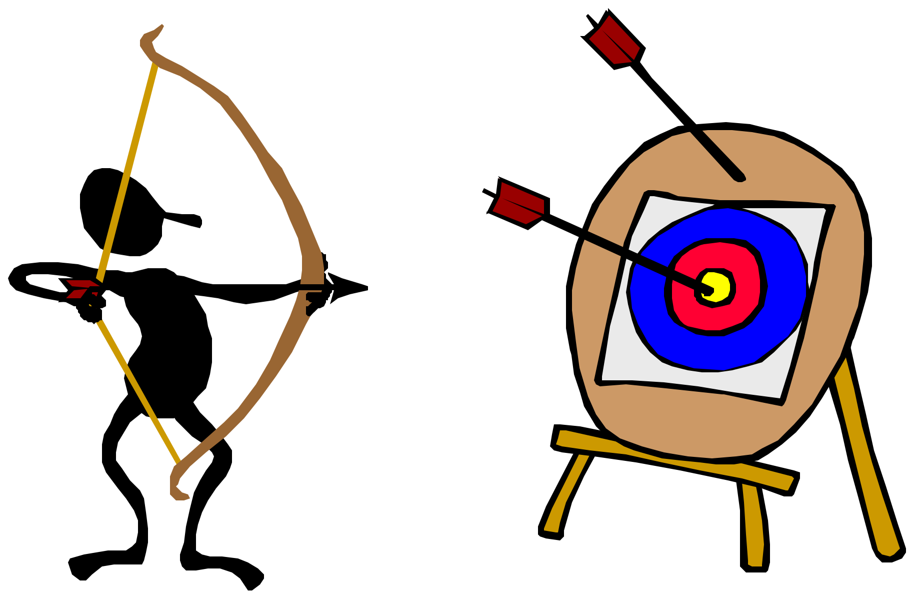
$$\log a = -1.204 = const$$

$$b = 1.5$$

$$l = .10$$

$$\Downarrow$$

$$T_{75} = -1.217$$



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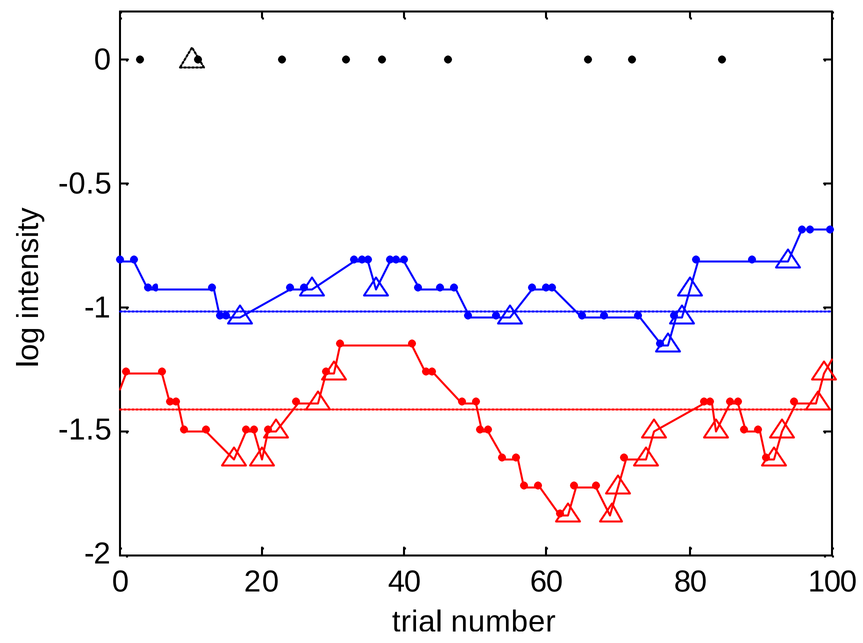
Stimulus Placement

- 2 interleaved staircases
- 100 trials/block
  - 10 catch
  - 40 x 3down/1up
  - 50 x 2down/1up
- 100 runs of 12 blocks each



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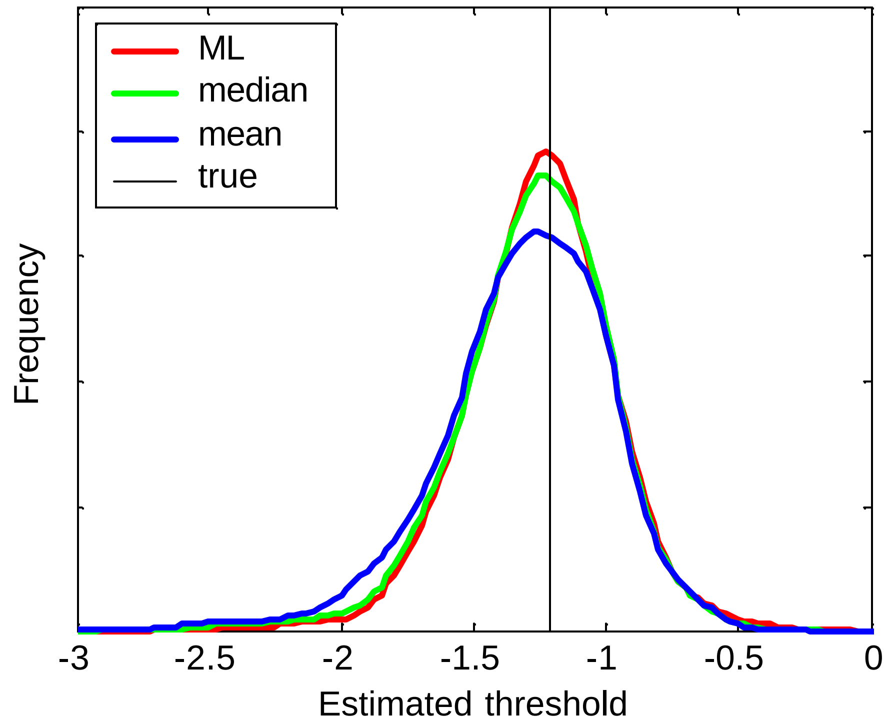
Threshold Estimators

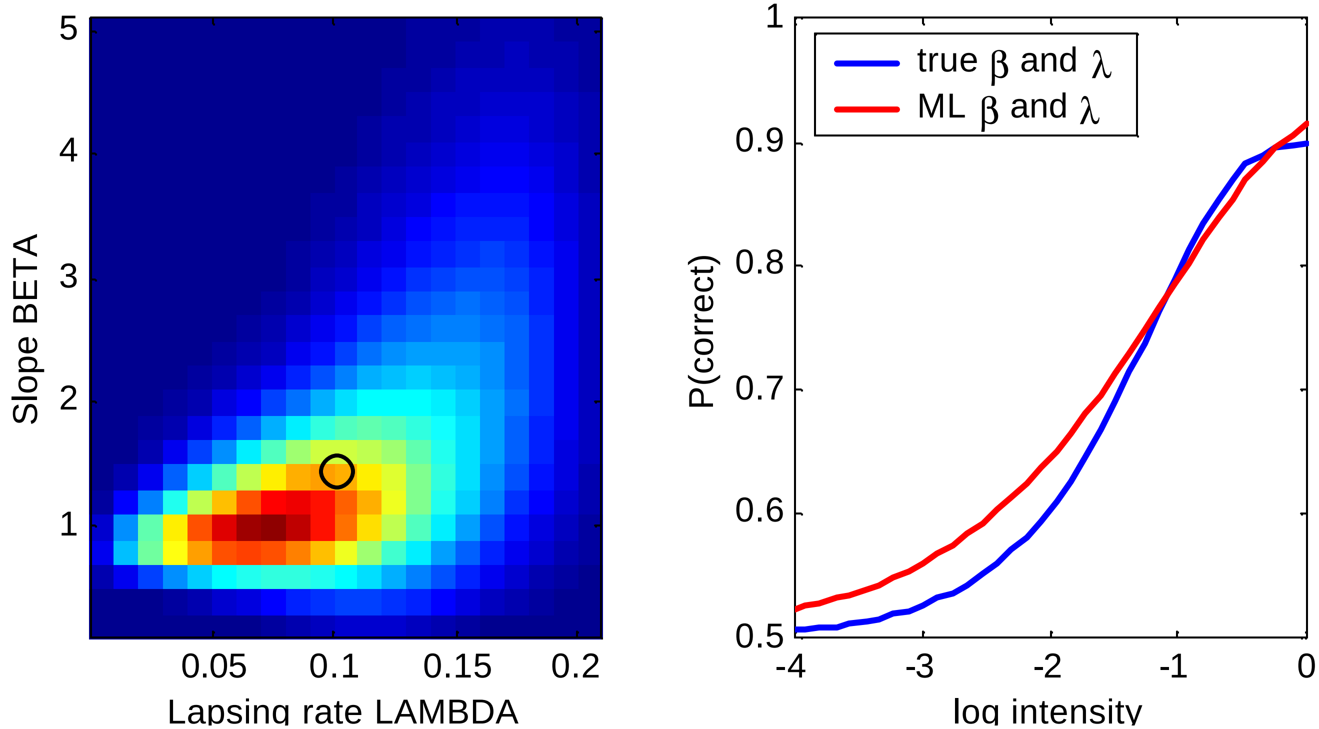
Estimator	Mean	Med	Std
ML	-1.24	-1.23	.27
Median	-1.26	-1.23	.28
Mean	-1.30	-1.27	.31
Std. dev.	0.41	0.36	.15



1200 Monte Carlo estimates

True 75% threshold = -1.217

β x λ Distribution from Pass 1



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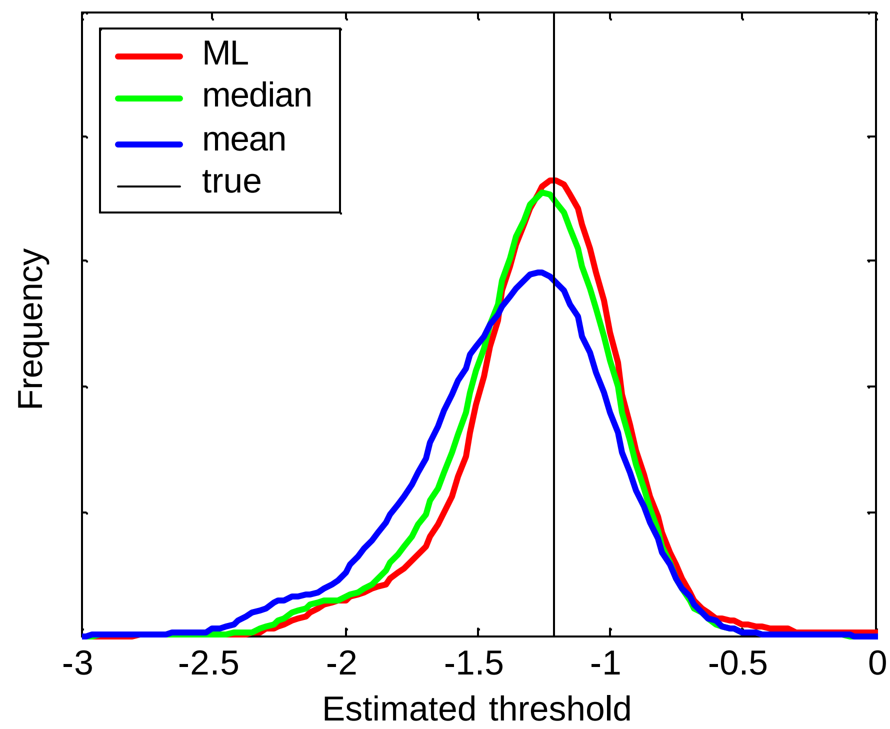
Catch Trials Are Worthwhile

Estimator	Mean	Med	Std
ML	-1.24	-1.22	.31
Median	-1.29	-1.26	.30
Mean	-1.36	-1.33	.34
Std. dev.	0.58	0.57	.16

1200 Monte Carlo estimates

No catch trials presented

True 75% threshold = -1.217

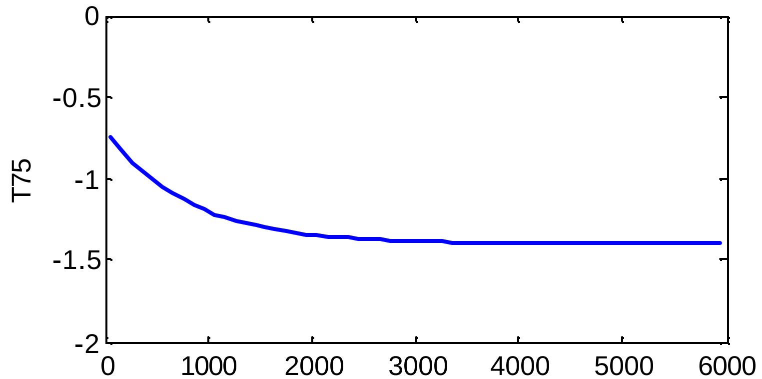


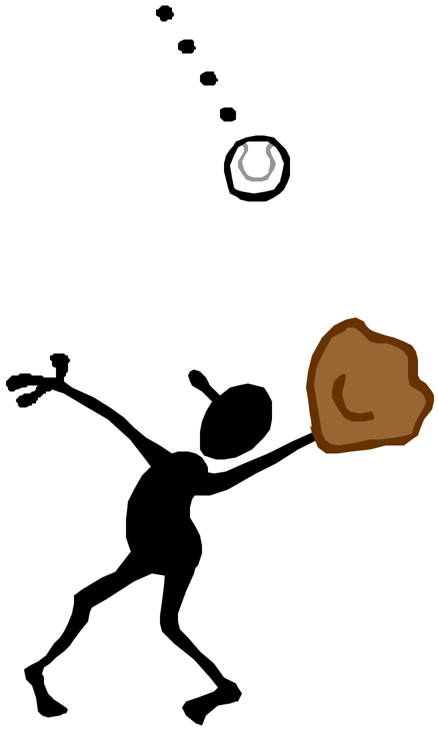
Simulation 2: With Learning

$$\log a = -0.693\,(e^{-t/800} - 2)$$

$$b = 1.5$$

$$l = .10$$





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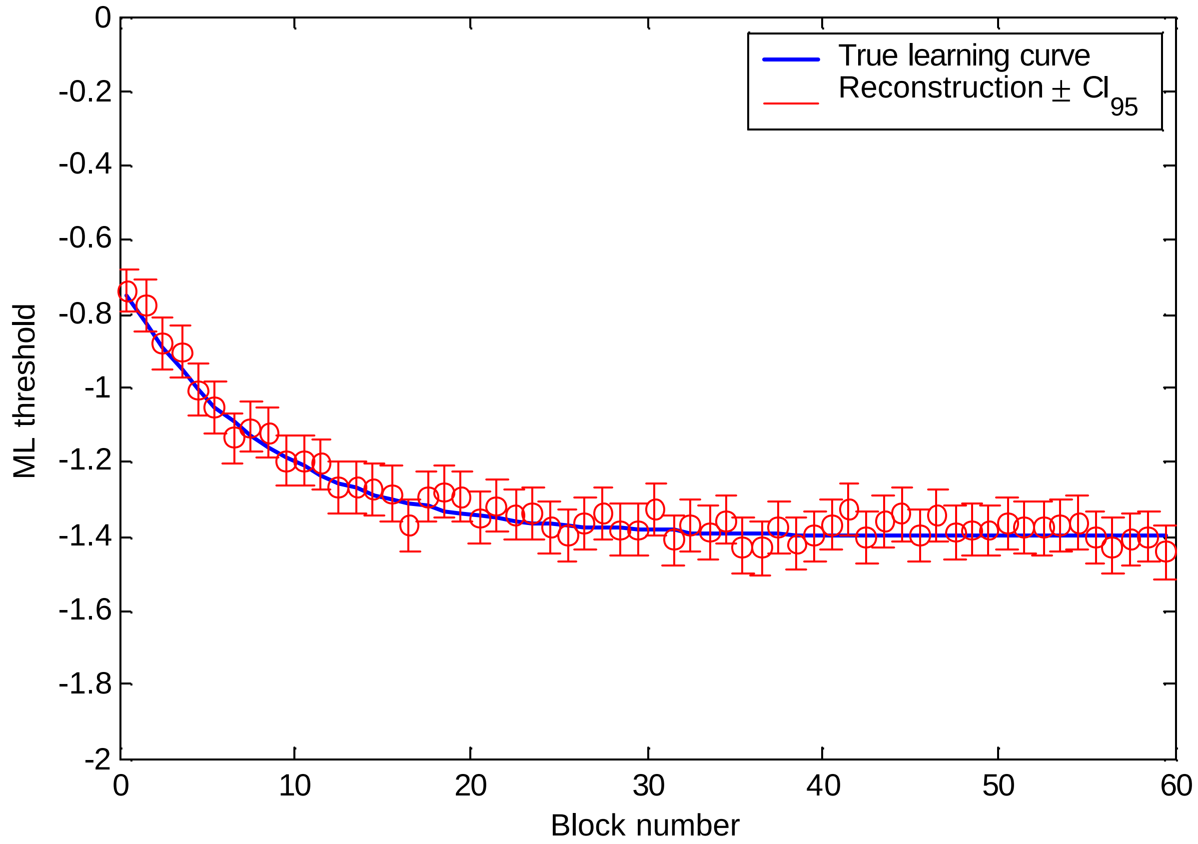
Group Learning Curve, N=100



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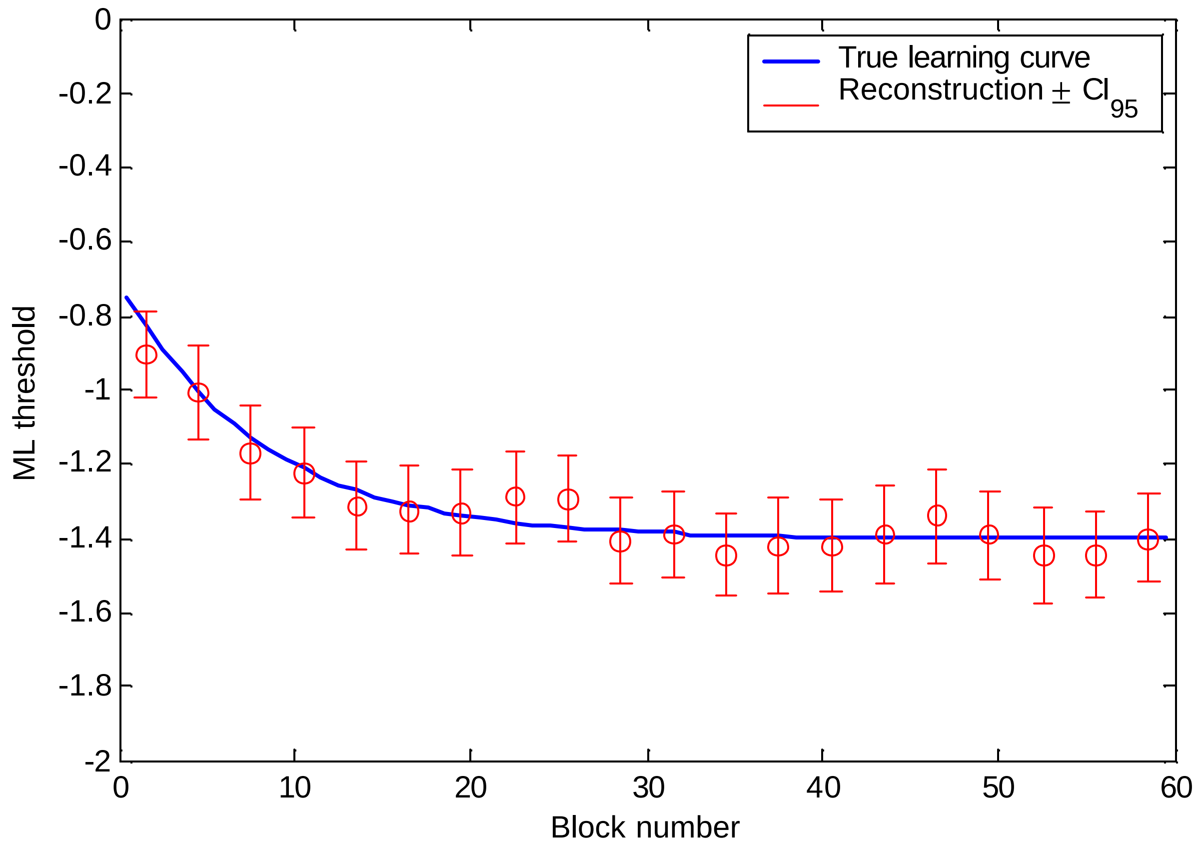
More Realistic Sample, N=10



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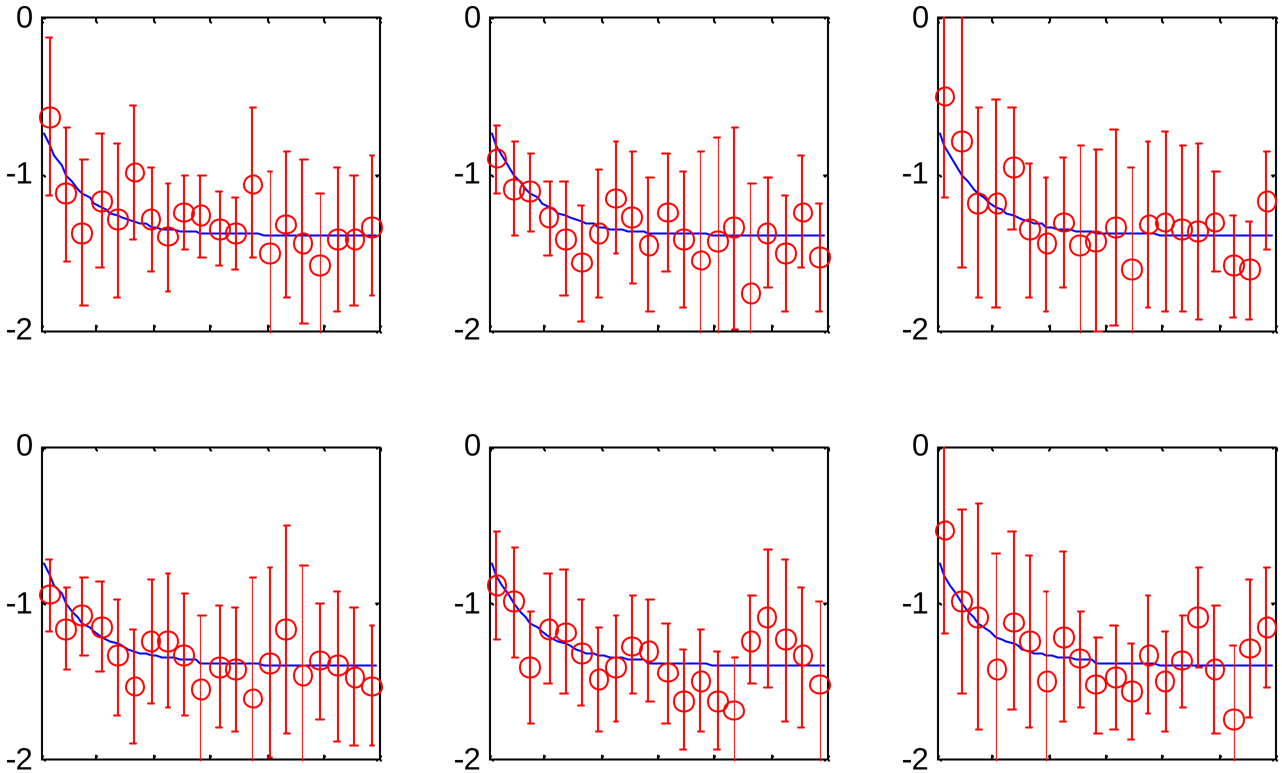
Individual Runs



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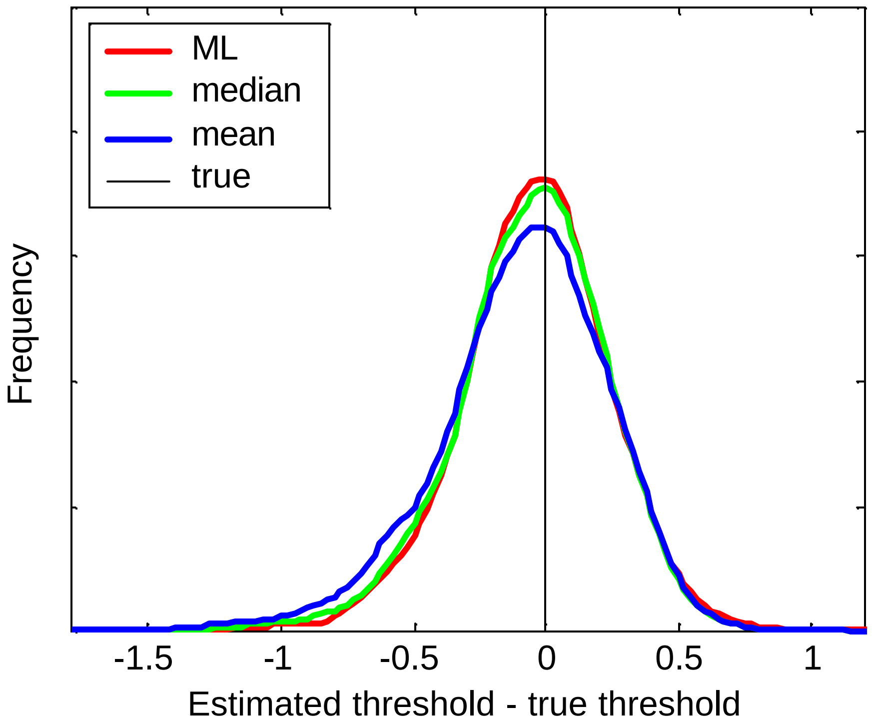
The Method Performs Well

Estimator	Mean	Med	Std
ML	-0.03	-0.02	.28
Median	-0.05	-0.03	.29
Mean	-0.08	-0.05	.32
Std. dev.	0.42	0.39	.15

6000 Monte Carlo estimates

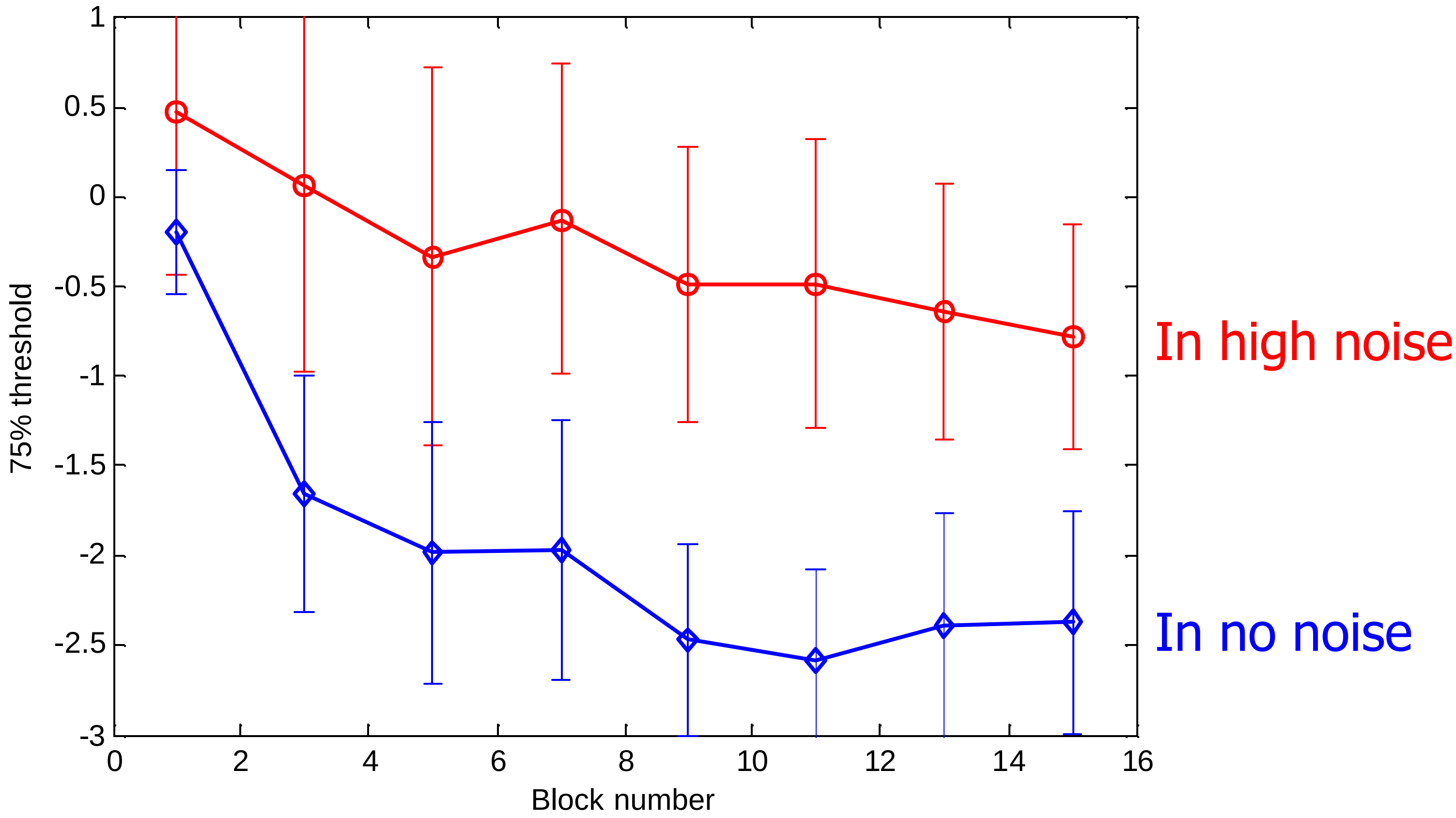
Similar to the stationary case

No systematic bias over time



Example: Actual Data, N=8

Jeter, Dosher, Petrov, & Lu (2005)



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Future Work

- Sensitivity to priors?
- Compare with standard ML methods
- Individual differences
- Estimate slope in addition to threshold
- Non-stationary β and λ?
- Recommended stimulus placement?
- Hierarchical models

The End

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