Bayesian Method for Repeated Threshold Estimation

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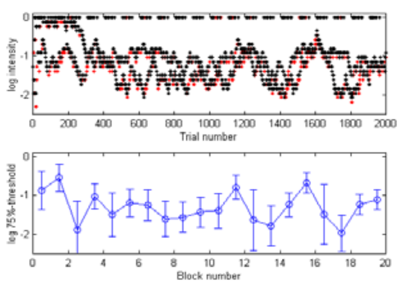
Motivation: Perceptual Learning

- Non-stationary thresholds
- Dynamics of learning is important
- Must use naïve observers
- Low motivation → high lapsing rates
- Slow learning → many sessions

- Large volume of low-quality binary data

Objective: Data Reduction



Isn't This a Solved Problem?

- Up/down (Levitt, 1970)
- PEST (Taylor & Creelman, 1967)
- BEST PEST (Pentland, 1980)
- QUEST (Watson & Pelli, 1979)
- ML-Test (Harvey, 1986)
- Ideal (Pelli, 1987)
- YAAP (Treutwein, 1989)
- and many others...

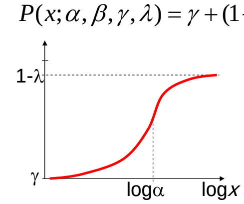
We Solve a Different Problem

- Standard methods:
  - Adaptive stimulus placement
  - Stopping criterion
  - Threshold estimation
- Our method:
  - Threshold estimation
  - Integrate information across blocks

Weibull Psychometric Function

$$W(x;\alpha,\beta) = 1 - \exp(-\exp((\log x - \log\alpha)\beta))$$

$$P(x;\alpha,\beta,\gamma,\lambda) = \gamma + (1-\gamma-\lambda)W(x;\alpha,\beta)$$



- Threshold log α
- Slope β
- Guessing rate γ
- Lapsing rate λ

Two Kinds of Parameters

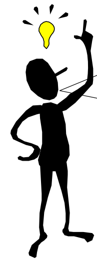
- Threshold log α
- Slope β
- Guessing rate γ
- Lapsing rate λ

$\left.\begin{array}{l}\text{Threshold log }\alpha\\ \text{Slope }\beta\end{array}\right\}$ Parameters of interest θ

$\left.\begin{array}{l}\text{Guessing rate }\gamma\\ \text{Lapsing rate }\lambda\end{array}\right\}$ Nuisance parameters ϕ

The nuisance parameters are harder to estimate but change more slowly than the threshold parameter.

Get the Best of Both Worlds



Use long data sequences to constrain the nuisance parameters; use short sequences to estimate the thresholds.

Joint Posterior of θ_k,ϕ

$$p(\theta_k,\phi \mid \mathbf{y}_k;\mathbf{y}_1\cdots\mathbf{y}_{k-1},\mathbf{y}_{k+1}\cdots\mathbf{y}_n) =$$

$$\underbrace{p(\mathbf{y}_k \mid \theta_k,\phi)}_{\text{Likelihood of current data}}\underbrace{\underbrace{p(\theta_k)\,p(\phi)}_{\text{Priors}}\underbrace{\prod_{i\neq k}\int p(\theta_i)\,p(\mathbf{y}_i \mid \theta_i,\phi)\,d\theta_i}_{\text{Information about }\phi\text{ extracted from the other data sets}}}_{\text{Modified prior for the current block}}$$

Two-Pass Algorithm

- Pass 1: for each block *i*, calculate

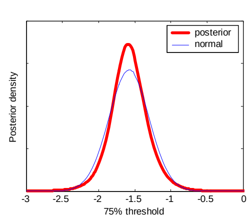
$$p(\phi \mid \mathbf{y}_i) = \int p(\theta)\,p(\mathbf{y}_i \mid \theta,\phi)\,d\theta$$

- Pass 2: for each block *k*, calculate

$$p(\theta_k,\phi \mid \mathbf{y}_k) = p(\mathbf{y}_k \mid \theta_k,\phi)\,p(\theta_k)\,p(\phi)\prod_{i\neq k}\int p(\phi \mid \mathbf{y}_i)$$

Posterior Thresholds

$$p(T_k) = \int P^{-1}(.75;\theta_k,\phi)\,p(\theta_k,\phi \mid \mathbf{y}_k)\,d\theta_k\,d\phi$$



Some Details

- Vaguely informative priors:

$$p(\log\alpha) \propto \mathrm{N}(\mu_\alpha,\sigma_\alpha)$$
$$p(\beta) \propto \mathrm{N}(\mu_\beta,\sigma_\beta)$$
$$p(\lambda) \propto \mathrm{Beta}(a_\lambda,b_\lambda)$$

- Implemented on a grid: logα x β x λ
- Assume γ=.5 for 2AFC data
- MATLAB software available at http://alexpetrov.com/softw/

Simulation 1: Stationary

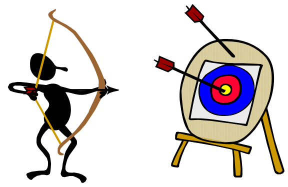
$\log \alpha = -1.204 = const$

$\beta = 1.5$

$\lambda = .10$

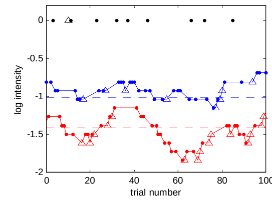
↓

$T_{75} = -1.217$



Stimulus Placement

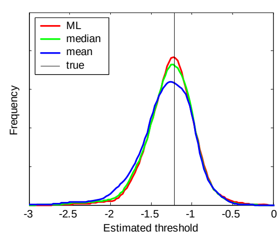
- 2 interleaved staircases
- 100 trials/block
  - 10 catch
  - 40 x 3down/1up
  - 50 x 2down/1up
- 100 runs of 12 blocks each



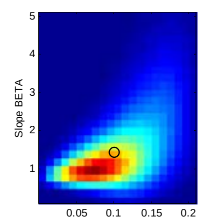
Threshold Estimators

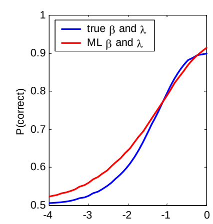
Estimator	Mean	Med	Std
ML	-1.24	-1.23	.27
Median	-1.26	-1.23	.28
Mean	-1.30	-1.27	.31
Std. dev.	0.41	0.36	.15

1200 Monte Carlo estimates
True 75% threshold = -1.217



β x λ Distribution from Pass 1

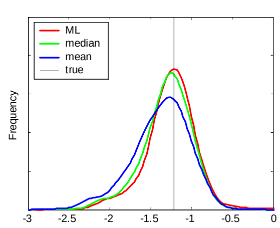




Catch Trials Are Worthwhile

Estimator	Mean	Med	Std
ML	-1.24	-1.22	.31
Median	-1.29	-1.26	.30
Mean	-1.36	-1.33	.34
Std. dev.	0.58	0.57	.16

1200 Monte Carlo estimates
No catch trials presented
True 75% threshold = -1.217

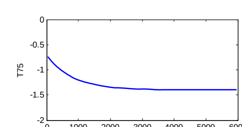


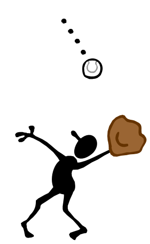
Simulation 2: With Learning

$\log \alpha = -0.693\,(e^{-t/800} - 2)$

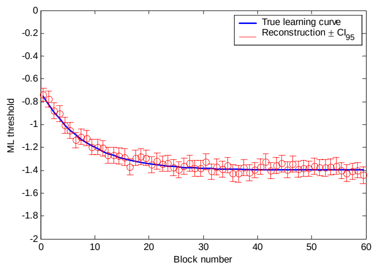
$\beta = 1.5$

$\lambda = .10$

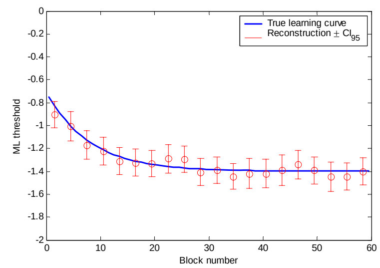




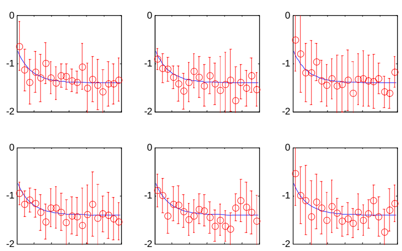
Group Learning Curve, N=100



More Realistic Sample, N=10



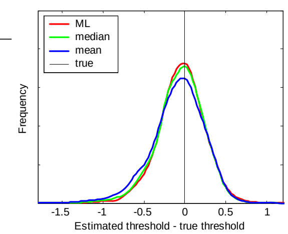
Individual Runs



The Method Performs Well

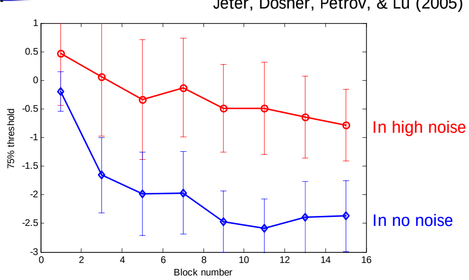
Estimator	Mean	Med	Std
ML	-0.03	-0.02	.28
Median	-0.05	-0.03	.29
Mean	-0.08	-0.05	.32
Std. dev.	0.42	0.39	.15

6000 Monte Carlo estimates
Similar to the stationary case
No systematic bias over time

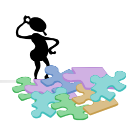


Example: Actual Data, N=8

Jeter, Dosher, Petrov, & Lu (2005)



Future Work



- Sensitivity to priors?
- Compare with standard ML methods
- Individual differences
- Estimate slope in addition to threshold
- Non-stationary β and λ?
- Recommended stimulus placement?
- Hierarchical models